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Toward a Standard Nomenclature for Earth–Mars Cycler Trajectories

T. Troy McConaghy*
Purdue University, West Lafayette, Indiana 47907-2023
Ryan P. Russell†
University of Texas at Austin, Austin, Texas 78712-1085
and
James M. Longuski‡
Purdue University, West Lafayette, Indiana 47907-2023

Many Earth–Mars cycler trajectories are now known and their numbers continue to grow. Unfortunately, the literature on Earth–Mars cycler trajectories uses many different systems for naming the various cyclers being investigated. A nomenclature is proposed as a remedy to standardize the naming system for near-ballistic Earth–Mars cycler trajectories. Modeling assumptions are given and the proposed nomenclature is explained. All known near-ballistic cyclers fall within the scope of the described nomenclature. Examples are presented of how several well-known cyclers are denoted. The syntax of the nomenclature is formally specified using the extended Backus–Naur form. Criteria for evaluating Earth–Mars cycler trajectories are summarized.

Nomenclature

a = orbit semimajor axis, km
c = chord length in the Lambert problem, km
$\boldsymbol{h}$ = angular momentum, kg · km^2/s
i' = orientation angle of half-revolution transfer plane, rad
K = number of Earth–Earth transfers per repeat interval
M = whole number of Earth revolutions on a transfer
N = whole number of spacecraft revolutions on a transfer
n = cycler repeat time in synodic periods
$\boldsymbol{r}$ = position vector, km
r = distance from the sun, km
S = Earth–Mars synodic period, years
s = semiperimeter of space triangle in Lambert problem, km
t_f = transfer time of flight, years
$\boldsymbol{v}$ = velocity, km/s
v = speed, km/s
$\boldsymbol{v}_\infty$ = spacecraft hyperbolic excess velocity, km/s
v_∞ = spacecraft hyperbolic excess speed, km/s
γ = flight-path angle, rad
ε = Lambert solution type (U, L, L_l or L_s)
θ = transfer angle, rad
λ = longitude angle of $\boldsymbol{v}_{\text{out}}$ on full-revolution transfers, rad
μ = gravitational parameter of the sun, km^3/s^2
φ = latitude angle of $\boldsymbol{v}_{\text{out}}$ on full-revolution transfers, rad

Subscripts

E = Earth
i = ith transfer leg
in = incoming (upon arrival at Earth)
out = outgoing (upon departure from Earth)
1 = spacecraft at the beginning of a transfer
2 = spacecraft at the end of a transfer

Introduction

EARTH–MARS cycler trajectories (cyclers) are trajectories that periodically have gravity-assist encounters with both Earth and Mars. A cycler vehicle (on these trajectories) carries people back and forth between Earth and Mars and could be massive because it would never stop at either Earth or Mars. (Small taxi spacecraft could ferry people from Earth up to the cycler vehicle or from the vehicle down to Mars and vice versa.)

Cyclers were first investigated by Rall[1] and Rall and Hollister.[2] Examples include the Aldrin cycler[3–5] and the Versatile International Station for Interplanetary Transport (VISIT) cyclers.[4,6–9] A recently renewed interest in cyclers led to the discovery of many new cyclers.[10–14] Landau and Longuski[15] have shown that Earth–Mars transportation systems using cyclers often compare favorably to other mission architectures (in terms of propellant required to deliver a given payload to Mars' surface).

Unfortunately, the various investigators used different naming systems (nomenclatures) for the Earth–Mars cyclers that they considered. Moreover, each nomenclature was tailored to the special subset of cyclers under consideration.

A common nomenclature facilitates scientific communication. Because there are many cyclers known today, by giving the known cyclers standard names, we are better able to see their similarities and their differences. A nomenclature could help organize the known cyclers into groups or patterns that are easier to conceptualize. Finally, by constructing a general nomenclature for Earth–Mars cyclers, we may reveal the existence of cyclers that have not yet been investigated. (Often, one does not think of something until there is a name for it. Language and thought are closely coupled.)

In this paper, we propose a fairly general nomenclature for Earth–Mars cycler trajectories. We have designed the proposed nomenclature so that the cyclers described by it can be constructed and analyzed easily without having to solve a more general problem first. We give examples of how some well-known cyclers are labeled within our proposed nomenclature and we conclude with a discussion of criteria for evaluating cyclers.

Modeling Assumptions

Our proposed nomenclature for Earth–Mars cyclers is based on simplified models for the trajectories of Earth, Mars, and the spacecraft. We make the following modeling assumptions:

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*Ph.D. Candidate, School of Aeronautics and Astronautics; mcconagh@ecn.purdue.edu. Student Member AIAA.

†Ph.D. Candidate, Department of Aerospace Engineering and Engineering Mechanics; ryanrussell@mail.utexas.edu. Student Member AIAA.

‡Professor, School of Aeronautics and Astronautics; longuski@ecn.purdue.edu. Associate Fellow AIAA.

1) To simplify the repeatability conditions for cyclers, we assume that the orbits of the planets are circular and coplanar.

2) The only significant gravitating bodies are Earth and the sun (for example, Mars may be encountered on a leg, but the encounter does not change the orbit of the spacecraft).

3) All transfer legs are conic sections; that is, they are solutions to the sun–spacecraft two-body problem.

4) Gravity-assist maneuvers occur instantaneously. Equivalently, gravity-assist maneuvers are modeled with an impulsive Δv.

5) All transfer legs travel around the sun in the same direction as the planets; that is, all transfers are direct. We neglect retrograde transfers (including hyperbolic transfers) which are not useful in constructing practical cyclers due to their high v_∞ at Earth.

6) Propulsive maneuvers never occur in deep space, but they may occur at Earth encounters.

We note that we do not assume that the Earth–Mars synodic period S is exactly $2\frac{1}{7}$ years (or any other approximation). Also, we allow the spacecraft to make out-of-plane transfers. Our approach to constructing cyclers is identical to Poincaré's method of constructing "second species" periodic solutions to the circular restricted three-body problem when the mass of the secondary body (e.g., the mass of Earth) is zero.[16–18]

The model could be made more accurate, for example, by allowing for gravity-assist maneuvers at Mars or by using ephemeris data for the planets. We have found that cyclers that exist within the simplified model often correspond to cyclers in a more accurate model.[5,14] In this paper, we use the simplified model only.

At the end of this paper, we show how the proposed nomenclature can be modified to allow for non-Earth–Earth transfers and gravity-assist maneuvers at bodies other than Earth. (In other words, we show how our nomenclature can be extended to allow for the removal of assumption 2.) For this reason, all equations will be given for the general case where the initial and final radii of the spacecraft (r_1 and r_2) are not necessarily equal.

Proposed Nomenclature

As explained by McConaghy et al.,[10] all Earth–Mars cyclers repeat after an integer number of Earth–Mars synodic periods. We will use the letter n to denote the repeat time in synodic periods, and so n can have the values 1, 2, 3, and so on.

Once the repeat time is given, the specification of the cycler is completed by describing each Earth–Earth transfer leg. Therefore, we propose giving all cyclers (constructed within the assumptions of our model) a "cycler label" of the following form:

$$nd_1d_2\cdots d_K$$

where d_i is a description string for the ith Earth–Earth transfer leg and K is the number of Earth–Earth transfer legs that occur every nS years (the repeat time of the cycler). We will refer to the d_i as "leg descriptors."

Each Earth–Earth transfer leg can be one of three different types, depending on the transfer angle θ (which can be greater than 2π rad for multiple-revolution transfers). A "half-revolution transfer" takes the spacecraft to the opposite side of the sun (in possibly more than one revolution of the sun). A "full-revolution transfer" takes the spacecraft back to where it started (in one or more revolutions of the sun). All other transfers are "generic transfers." The leg descriptors for these three different types of transfer are now explained.

Generic Transfers

If a leg is a generic transfer, then we use a leg descriptor (d_i) of the form $g(t_f, \theta, \varepsilon)$ where the g stands for "generic." The first parameter, t_f, is the transfer time of flight, in years. For Earth–Earth transfers, t_f is also equal to the number of revolutions Earth makes around the sun during the transfer. The second parameter, θ, is the transfer angle. For multiple-revolution transfers, $\theta > 2\pi$ rad.

The third parameter, ε, requires a bit more explanation. The determination of the transfer orbit's semimajor axis a from t_f and θ is a Lambert problem. [We note that, for Earth–Earth transfers, $r_1 = r_2 = r_E = 1$ astronomical unit (AU).] If the transfer angle θ is greater than 2π rad, then there are two different possible transfers with the same t_f and θ, but with a different a. (There are also retrograde transfers, but we ignore them.) Figure 1 illustrates some specific examples. Each point on Fig. 1 corresponds to an Earth–Earth transfer with a transfer angle of $\theta = \pi/2 + 2\pi N$ rad (where N is the whole number of spacecraft revolutions). For example, the $N = 2$ curve corresponds to transfers with $\theta = 9\pi/2$ rad. We see that, if $t_f = 2.5$ years and $\theta = 9\pi/2$ rad, there are two different possible values of a. The ε parameter is required to indicate which of the two possible a values is actually used.

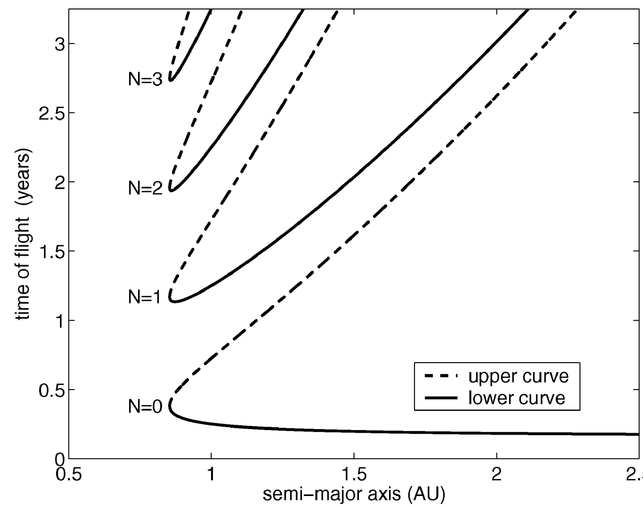

Fig. 1 Earth–Earth transfers with $\theta = \pi/2 + 2\pi N$ rad.

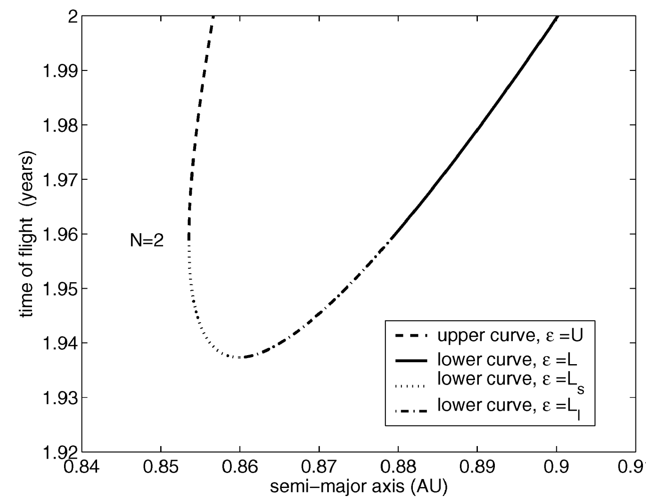

Fig. 2 Four possible values of ε.

Because ε indicates which of the two a values actually gets used, it only needs to have two possible values. For example, ε might be L on the left solution and R on the right solution. Shen and Tsiotras[19] used this approach.

However, there are several different ways to formulate and solve a multiple-revolution Lambert problem. To allow for the use of the classical Lagrange formulation, we let ε take four possible values: U, L, L_s, and L_l. The meanings of these values are illustrated in Fig. 2, which zooms in on the $N = 2$ curve of Fig. 1. We note that the minimum-energy transfer corresponds to the point where the $\varepsilon = U$ curve meets the $\varepsilon = L_s$ curve. Similarly, the minimum time-of-flight transfer corresponds to the point where the $\varepsilon = L_s$ curve meets the $\varepsilon = L_l$ curve.

If one uses the Lagrange formulation of Lambert's problem, then the equation to solve for a is[20]

$$\sqrt{\mu}t_f = a^{\frac{3}{2}}[2\pi N + \alpha - \beta - \sin(\alpha) + \sin(\beta)] \tag{1}$$

where μ is the gravitational parameter of the sun, N is the number of spacecraft revolutions, rounded down to the nearest integer, that is,

$$N = \text{floor}(\theta/2\pi) \tag{2}$$

$$\alpha = \begin{cases} \alpha_0 & \text{if} \quad \varepsilon = L, L_l, \text{ or } L_s \\ 2\pi - \alpha_0 & \text{if} \quad \varepsilon = U \end{cases} \tag{3}$$

$$\beta = \begin{cases} \beta_0 & \text{if} \quad 0 < [\theta \pmod{2\pi}] < \pi \\ -\beta_0 & \text{if} \quad \pi < [\theta \pmod{2\pi}] < 2\pi \end{cases} \tag{4}$$

$$\alpha_0 = 2 \arcsin \sqrt{s/(2a)} \tag{5}$$

$$\beta_0 = 2 \arcsin \sqrt{(s-c)/(2a)} \tag{6}$$

$$s = (r_1 + r_2 + c)/2 \tag{7}$$

$$c = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos\theta} \tag{8}$$

In Eq. (3), we see that ε is used to determine the value of α. If $\varepsilon = U$, then the solution is on the upper curve and $\alpha = 2\pi - \alpha_0$. If $\varepsilon = L$, L_l, or L_s, then the solution is on the lower curve and $\alpha = \alpha_0$.

Sometimes there are two solutions on the lower curve that have the same t_f but different a. In these special cases, the extra subscript l on L_l indicates that the long-period solution should be used and the subscript s on L_s indicates that the short-period solution should be used. Prussing[20] explains how to handle this case in greater detail.

One may choose to use other methods to determine a (such as Battin's method[19,21] or Lancaster's method[22]). For those methods, if $\varepsilon = U$ or L_s, the transfer is the left-hand, short-period solution, and if $\varepsilon = L$ or L_l, then the transfer is the right-hand, long-period solution.

Once a has been determined, the outgoing velocity of the spacecraft at Earth departure, $\boldsymbol{v}_{\text{out}}$, can be calculated using[23]

$$\boldsymbol{v}_{\text{out}} = [(B+A)/c](\boldsymbol{r}_2 - \boldsymbol{r}_1) + [(B-A)/r_1]\boldsymbol{r}_1 \tag{9}$$

where

$$A = \sqrt{\mu/4a}\cot(\alpha/2) \tag{10}$$

$$B = \sqrt{\mu/4a}\cot(\beta/2) \tag{11}$$

The incoming velocity of the spacecraft upon return to Earth, $\boldsymbol{v}_{\text{in}}$, can also be calculated[23]:

$$\boldsymbol{v}_{\text{in}} = [(B+A)/c](\boldsymbol{r}_2 - \boldsymbol{r}_1) - [(B-A)/r_2]\boldsymbol{r}_2 \tag{12}$$

Full-Revolution Transfers

If a leg is a full-revolution transfer, then we use a leg descriptor (d_i) of the form $f(M : N, \varphi, \lambda)$ where the f stands for "full-revolution." The first parameter, M, is the number of Earth revolutions, and so for Earth–Earth transfers, M also equals the transfer time of flight in years. The second parameter, N, is the number of spacecraft revolutions made during the transfer. (We note that the ratio M/N conveniently provides the spacecraft orbit period in Earth years.) Both M and N are integers. Because the transfer time is M Earth-orbit periods and N spacecraft-orbit periods,

$$M \cdot 2\pi\sqrt{a_E^3/\mu} = N \cdot 2\pi\sqrt{a^3/\mu} \tag{13}$$

where a_E is the semimajor axis of Earth's orbit (1 AU). Because of this resonance property, full-revolution transfers are also known as resonant transfers. For example, if Earth makes three revolutions while the spacecraft makes two revolutions, then the transfer is known as a 3:2 resonant transfer (and the spacecraft has an orbit period of $1\frac{1}{2}$ years). Equation (13) can be solved for a:

$$a = a_E(M/N)^{\frac{2}{3}} \tag{14}$$

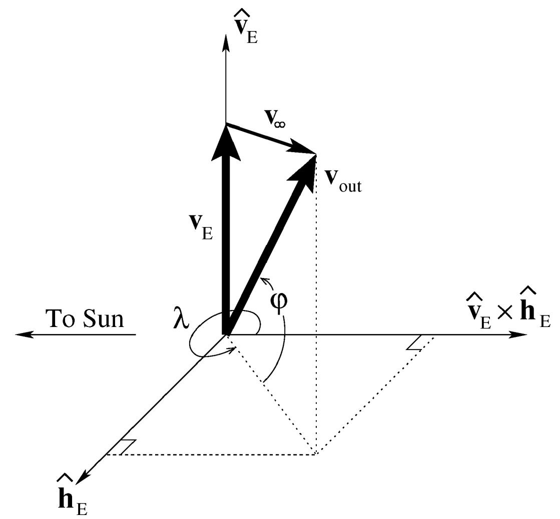

Fig. 3 Definition of the angles φ and λ for Earth departure.

We note that M and N are constrained because the distance from the transfer orbit's aphelion to its perihelion ($2a$) must be greater than the distance between Earth and the sun (a_E). Hence, $a > a_E/2$. The magnitude of the spacecraft's outgoing velocity, v_{out}, can be determined from a using the vis viva equation:

$$v_{\text{out}} = \sqrt{\mu(2/r_E - 1/a)} \tag{15}$$

where r_E is the orbital radius of Earth (1 AU). The spacecraft's outgoing velocity after the Earth encounter, $\boldsymbol{v}_{\text{out}}$, can lie anywhere on the surface of a sphere of radius v_{out}. The angles φ and λ are used to indicate the direction of $\boldsymbol{v}_{\text{out}}$ with respect to a rotating reference frame as shown in Fig. 3. The unit vector $\hat{\boldsymbol{v}}_E$ is in the direction of the Earth's velocity vector ($\boldsymbol{v}_E$) and $\hat{\boldsymbol{h}}_E$ is a unit vector in the direction of Earth's orbital angular momentum vector ($\boldsymbol{h}_E$). The third direction, $\hat{\boldsymbol{v}}_E \times \hat{\boldsymbol{h}}_E$, completes the right-handed reference frame. When Earth is assumed to be in a circular orbit, $\hat{\boldsymbol{v}}_E \times \hat{\boldsymbol{h}}_E$ is in the direction from the sun to Earth; that is, $\hat{\boldsymbol{v}}_E \times \hat{\boldsymbol{h}}_E = \hat{\boldsymbol{r}}_E$.

We see that φ is a latitude angle (between -90 and 90 deg) and λ is a longitude angle (unconstrained). These two angles are convenient for expressing the degrees of freedom because the v_∞, an important metric for cyclers, is a function of φ (using the law of cosines) and is independent of λ. The vector $\boldsymbol{v}_{\text{out}}$ can be written as

$$\boldsymbol{v}_{\text{out}} = v_{\text{out}}[(\cos\varphi\cos\lambda)(\hat{\boldsymbol{v}}_E \times \hat{\boldsymbol{h}}_E) - (\cos\varphi\sin\lambda)\hat{\boldsymbol{h}}_E + (\sin\varphi)\hat{\boldsymbol{v}}_E] \tag{16}$$

For a full-revolution transfer to be posigrade, φ must be between 0 and 90 deg. We note that $\boldsymbol{v}_{\text{out}}$ is above the ecliptic plane if λ is between -180 and 0 deg, and below the ecliptic plane if λ is between 0 and 180 deg.

Half-Revolution Transfers

If a leg is a half-revolution transfer, then we use a leg descriptor (d_i) of the form $h(t_f, N, \varepsilon, i')$. The h stands for "half-revolution," and t_f, N, and ε mean the same as in the leg descriptor of a generic transfer.

The fourth parameter, i', is needed to specify the orientation of the transfer orbit plane (which is free because the Earth departure point, the sun, and the Earth arrival point all lie on a line). Specifically, i' is the signed angle between the transfer orbit's angular momentum, $\boldsymbol{h} = \boldsymbol{r}_E \times \boldsymbol{v}_{\text{out}}$, and Earth's orbital angular momentum ($\boldsymbol{h}_E$). The sign of i' is always the same as the sign of ($\boldsymbol{v}_{\text{out}} \cdot \hat{\boldsymbol{h}}_E$). For the transfer to be posigrade, i' must be between -90 and 90 deg. We note that i' is not the same as the inclination i of the transfer orbit although $i' = i$ when i' is between 0 and 180 deg (hence the need for the prime on i'). When a half-revolution transfer with $N = 0$ follows a gravity-assist maneuver, it is called a backflip.[24] A backflip with $i' > 0$ deg is known as a northern backflip and a backflip with $i' < 0$ deg is known as a southern backflip.

To determine a from t_f, N, and ε, one uses the same procedure as with generic transfers. The spacecraft's outgoing velocity can then be calculated using[13,21]

$$\boldsymbol{v}_{\text{out}} = (\dot{r}_{\text{out}})\hat{\boldsymbol{r}}_E + (\kappa_{\text{out}} \sin i')\hat{\boldsymbol{h}}_E + (\kappa_{\text{out}} \cos i')\hat{\boldsymbol{\theta}}_E \tag{17}$$

where

$$\dot{r}_{\text{out}} = \sqrt{\mu\left[\frac{2}{(r_1 + r_2)} - \frac{1}{a}\right]} \cdot \begin{Bmatrix} -1 & \text{if} & \varepsilon = L, L_l, \text{or } L_s \\ +1 & \text{if} & \varepsilon = U \end{Bmatrix} \tag{18}$$

$$\kappa_{\text{out}} = \sqrt{2\mu r_2 \big/ \left(r_1^2 + r_1 r_2\right)} \tag{19}$$

$$\hat{\boldsymbol{r}}_E = (\cos \gamma_E)(\hat{\boldsymbol{v}}_E \times \hat{\boldsymbol{h}}_E) + (\sin \gamma_E)\hat{\boldsymbol{v}}_E \tag{20}$$

$$\hat{\boldsymbol{\theta}}_E = (-\sin \gamma_E)(\hat{\boldsymbol{v}}_E \times \hat{\boldsymbol{h}}_E) + (\cos \gamma_E)\hat{\boldsymbol{v}}_E \tag{21}$$

and γ_E is Earth's flight-path angle. The spacecraft's incoming velocity at the end of the half-revolution transfer can be calculated using

$$\boldsymbol{v}_{\text{in}} = (\dot{r}_{\text{in}})\hat{\boldsymbol{r}}_{E2} - (\kappa_{\text{in}} \sin i')\hat{\boldsymbol{h}}_{E2} + (\kappa_{\text{in}} \cos i')\hat{\boldsymbol{\theta}}_{E2} \tag{22}$$

where

$$\dot{r}_{\text{in}} = -\dot{r}_{\text{out}} \tag{23}$$

$$\kappa_{\text{in}} = \sqrt{2\mu r_1 \big/ \left(r_2^2 + r_1 r_2\right)} \tag{24}$$

$$\hat{\boldsymbol{r}}_{E2} = -\hat{\boldsymbol{r}}_E \tag{25}$$

$$\hat{\boldsymbol{h}}_{E2} = \hat{\boldsymbol{h}}_E \tag{26}$$

$$\hat{\boldsymbol{\theta}}_{E2} = \hat{\boldsymbol{h}}_{E2} \times \hat{\boldsymbol{r}}_{E2} \tag{27}$$

The leg descriptors for the three types of transfer have now been explained. Table 1 gives a summary.

Discussion and Examples

To be consistent, the sum of the transfer leg durations in a cycler label must equal the repeat time (n times the Earth–Mars synodic period S); that is,

$$t_{f1} + t_{f2} + \cdots + t_{fK} = nS \tag{28}$$

where t_{fi} is the time of flight on the ith transfer leg.

Table 1 Transfer leg descriptors

Transfer type	Leg descriptor
Generic	$g(t_f, \theta, \varepsilon)$
Full-revolution	$f(M : N, \varphi, \lambda)$
Half-revolution	$h(t_f, N, \varepsilon, i')$

One might think that if there are K transfer legs then there are $(K - 1)$ intermediate Earth encounters, but that is not necessarily true. For example, suppose that a half-revolution transfer leg takes 1.5 years and has a period of 1 year. Then the spacecraft will encounter the Earth every half-year (assuming the transfer leg is inclined and the spacecraft doesn't use the intermediate Earth encounters for gravity-assist maneuvers). If Earth encounters at the beginning and the end of the leg are counted, then the leg has four Earth encounters. In cases like this, where a leg has hidden intermediate Earth encounters, we recommend breaking the leg into separate Earth–Earth transfers.

We note that, in our nomenclature, every cycler with K transfer legs can have up to K different cycler labels (because each of the K legs could be chosen as the first leg). Much of this repetition can be removed if the longest generic leg is selected to appear first.

Sometimes a transfer leg is repeated k times in a row. Rather than repeating its leg descriptor k times, we put a superscript k after the leg descriptor. For example, the superscript 3 in g (0.493, 121 deg, $U)^3$ means the leg is repeated three times. Table 2 shows the cycler labels of some well-known cyclers.

Figure 4 shows a plot of Cycler 2.5.1.+0 in an inertial frame. Cycler 2.5.1.+0 repeats every two synodic periods and uses all three kinds of transfer (generic, full-revolution, and half-revolution).

So far, we have only considered cyclers constructed out of Earth–Earth transfers. Other transfers such as Earth–Mars transfers or Venus–Venus transfers were not considered. Our proposed nomenclature is designed so that it also works for such non-Earth–Earth transfers. All that needs to be added is some indication of the planets visited during the cycle. For example, if a cycler leaves Earth and then encounters Mars, Venus, Venus, and Earth in that order, one could add the string EMVVE at the front of cycler label. All equations in this paper are given in a form general enough to make calculations for transfers between planets at different radii.

Although our proposed nomenclature is designed for Earth–Mars cycler trajectories, the leg descriptors could be used to describe almost any ballistic transfer, even if the leg is not part of a cycler

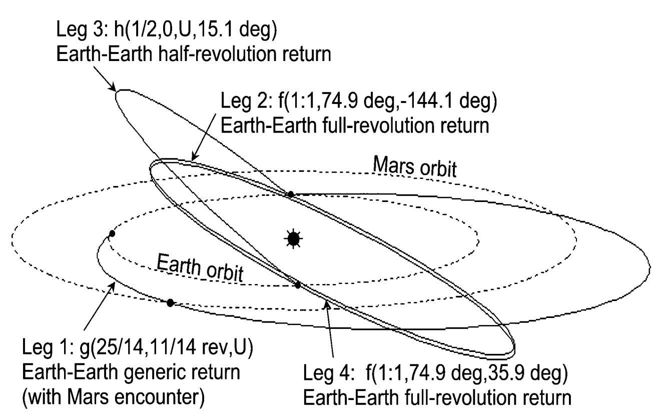

Fig. 4 Cycler 2.5.1.+0 (out-of-plane component expanded to make inclinations easier to see).

Table 2 Some well-known cyclers

Common name	References	Cycler label[a]
Aldrin cycler	3–5	$1g(2\frac{1}{7}, 1\frac{1}{7}$ rev, $L)$
VISIT 1 cycler	4, 6–9	$7f(5{:}4, \varphi, 0\text{ deg})^3$ or $7f(5{:}4, \varphi, 180\text{ deg})^3$
VISIT 2 cycler	4, 6–9	$7f(3{:}2, \varphi, 0\text{ deg})^5$ or $7f(3{:}2, \varphi, 180\text{ deg})^5$
Ballistic S1L1 cycler	10, 14	$2g(2.8277, 657.97\text{ deg}, U)\ g(1.4508, 522.29\text{ deg}, L)$
Byrnes' case 3 cycler	11	$2g(2\frac{11}{14}, 1\frac{11}{14}$ rev, $U)\ f(1{:}1, 79.612\text{ deg}, \lambda)\ldots$ $h(0.5, 0, \text{U}, \pm 10.388\text{ deg})$
Cycler 2.5.1.+0	12	$2g(1\frac{11}{14}, \frac{11}{14}$ rev, $U)\ f(1{:}1, 74.919\text{ deg}, \mp 144.069\text{ deg})\ldots$ $h(0.5, 0, \text{U}, \pm 15.081\text{ deg})\ldots$ $f(1{:}1, 74.919\text{ deg}, \pm 35.931\text{ deg})$
Cycler 4.3.1.−5	12	$4g(7\frac{1}{14}, 5\frac{1}{14}$ rev, $L)\ f(1{:}1, 84.039\text{ deg}, \mp 90.0\text{ deg})\ldots$ $h(0.5, 0, U, \pm 5.961\text{ deg})$

[a]Whenever a variable like φ or λ appears instead of a value, it is a free design variable.

Table 3 Constructing cycler labels

Label element[a]	Form[b]
⟨cycler label⟩	[(⟨body sequence⟩)]n⟨leg sequence⟩
⟨leg sequence⟩	⟨leg descriptor⟩ | ⟨leg descriptor⟩⟨leg sequence⟩
⟨leg descriptor⟩	$g(t_f, \theta, \varepsilon)^{[k]} \mid f(M : N, \varphi, \lambda)^{[k]} \mid h(t_f, N, \varepsilon, i')^{[k]}$
⟨body sequence⟩	⟨body abbreviation⟩⟨body sequence⟩ | ⟨body abbreviation⟩⟨body abbreviation⟩
⟨body abbreviation⟩	E|M|V| · · ·

[a]Also called a nonterminal.
[b]Also called a production rule. The pipe, |, means "or," and items in square brackets are optional.

trajectory. Extensions would have to be made to allow for retrograde, parabolic, hyperbolic, or rectilinear transfers (which never occur in cyclers).

Formal Specification of the Proposed Nomenclature

Cycler labels within the proposed nomenclature have a very specific syntax. A syntax can be formally specified using the extended Backus–Naur form (EBNF), a metalanguage commonly used for specifying the syntax of computer programming languages.[25] The EBNF specification of the proposed nomenclature is given in Table 3.

For example, the first row in Table 3 says that a ⟨cycler label⟩ may (optionally) begin with a ⟨body sequence⟩ in parentheses. That is followed by n (the repeat time in synodic periods), which is followed by a ⟨leg sequence⟩. The second row says (recursively) that a ⟨leg sequence⟩ is a sequence of one or more ⟨leg descriptor⟩ elements. Similarly, the fourth row says (recursively) that a ⟨body sequence⟩ is a sequence of two of more ⟨body abbreviation⟩ elements.

Evaluating Cyclers

When using the nomenclature described in this paper, the cycler label completely determines all characteristics of the cycler. If at least one of the Earth–Earth transfer legs crosses the orbit of Mars, then there are a number of important cycler characteristics that should be evaluated: 1) the number of vehicles needed to provide a short Earth–Mars and short Mars–Earth leg every synodic period should be as small as possible, 2) for each of the short transit legs between Earth and Mars (or between Mars and Earth), the transit time and the v_∞ at the two encounters should be as small as possible, and 3) the total required Δv should be zero, ideally. That is, the cycler should be a ballistic cycler. If some Δv is required to make up for insufficient gravity assist, for example, then it should be as small as possible. (Cyclers that require a nonzero Δv are known as powered cyclers.) Detailed explanations of these characteristics and the reasons for calculating them are given in other papers.[5,10–14]

These characteristics (number of vehicles, transit duration, v_∞, and Δv) can be combined in various ways to form a single cost function. Many different cost functions are possible. Alternatively, one might examine the tradeoffs between competing objectives (like short transfer time and low arrival v_∞).

Conclusions

We propose a nomenclature that is general enough to cover broad classes of Earth–Mars cyclers. The proposed system is designed to be complete, in the sense that a cycler's label determines all of its characteristics. The system is also designed so that the calculation of other characteristics (such as the semimajor axis) is relatively easy. Our hope is that future research on cyclers will use the proposed nomenclature (or a suitable generalization) to facilitate scientific communication and comparisons between different cyclers. We also hope that cyclers that have not been investigated, but which now have names within the nomenclature, will be investigated in the near future.

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C. Kluever
Associate Editor